Math 155, ***Lecture Notes- Bonds*** **Name____________**

<u>Section 10.2</u> *<u>Plane Curves and Parametric Equations</u>*

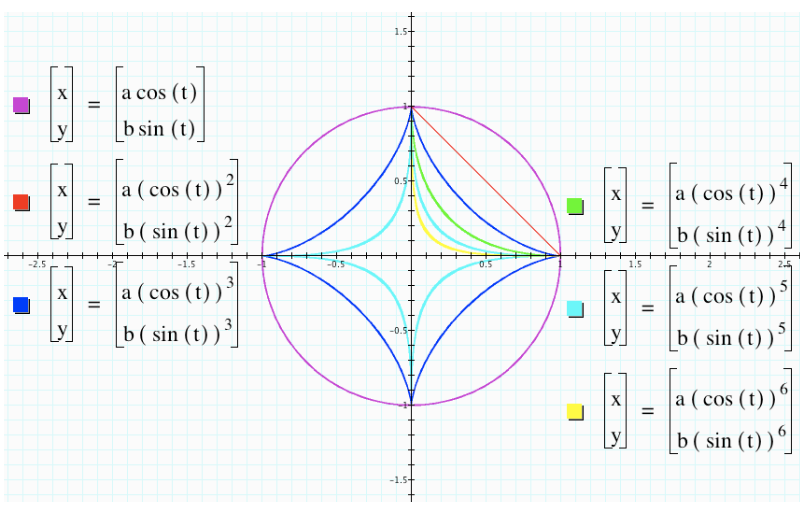
Until now, we have been studying curves that were represented by a single equation in two variables. In this section, we will consider curves that are defined using three variables, and these curves will be represented by a "system" of two equations in two variables, the one variable common to both equations is called the **parameter**. The "system" of two equations in two variables will be called **parameter equations**. In this section, we will write x as $x=x(t)$ and y as $y=y(t)$, x and y will both be functions of t, a parametric variable. At times, we will write x as $x=x(\theta)$ and y as $y=y(\theta)$, x and y will both be functions of θ, a parametric variable.

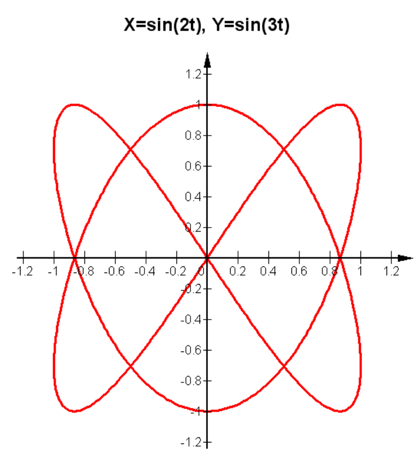
Definition of a Plane Curve

If f and g are continuous functions of t on an interval I, then the equations

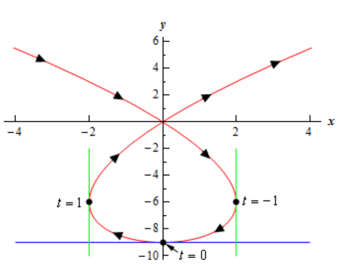
$$x = f(t) \quad \text{and} \quad y = g(t)$$

are called **parametric equations** and t is called the **parameter.** The set of points (x, y) obtained as t varies over the interval I is called the **graph** of the parametric equations. Taken together, the parametric equations and the graph are called a **plane curve,** denoted by C.

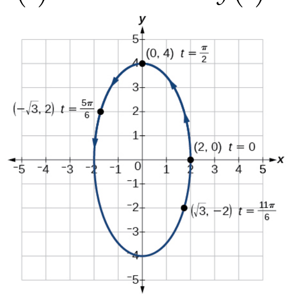




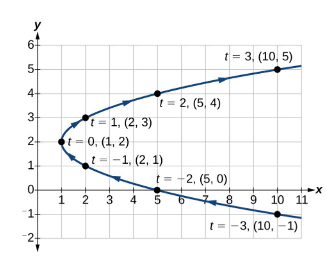
$x(t)=t^3-3t$ & $y(t)=3t^2-9$



$x(t)=2\cos t$ & $y(t)=4\sin t$



$x(t)=t^2+1$ & $y(t)=t+2$



Ex. 1: Sketch the curve represented by the parametric equations and write the corresponding rectangular equation by eliminating the parameter. Complete the table.

$$\begin{cases} x(t) = t^2 + t \\ y(t) = t^2 - t \end{cases} \text{ for } t \in [-3, 3]$$

t	x(t)	y(t)

More Ex. 1:

Ex. 2: Sketch the curve represented by the parametric equations and write the corresponding rectangular equation by eliminating the parameter. Use your graphing utility to confirm your result. Complete the table.

$$\begin{cases} x(t) = t^2 \\ y(t) = t \end{cases}$$

t	x(t)	y(t)

Ex. 3: Sketch the curve represented by the parametric equations and write the corresponding rectangular equation by eliminating the parameter. Use your graphing utility to confirm your result. Complete the table.

$$\begin{cases} x(\theta) = 3\cos(\theta) \\ y(\theta) = 4\sin(\theta) \end{cases} \text{ for } \theta \in \left[0, \frac{\pi}{2}\right]$$

θ	x(θ)	y(θ)

Ex. 4: Sketch the curve represented by the parametric equations and write the corresponding rectangular equation by eliminating the parameter. Use your graphing utility to confirm your result. Complete the table.

$$\begin{cases} x(t) = 3\sin(2t) \\ y(t) = 4\cos(2t) \end{cases} \quad \text{for } t \in \left[0, \frac{\pi}{2}\right]$$

t	x(t)	y(t)

Compare on your graphing utility:

$$\begin{cases} x(t) = 3\cos\left(\frac{\pi}{2} - 2t\right) \\ y(t) = 4\sin\left(\frac{\pi}{2} - 2t\right) \end{cases}$$

Ex. 5: Find a set of parametric equations for the line that passes through the points $(1,4)$ and $(5,-2)$ and write the corresponding rectangular equation by eliminating the parameter. Use your graphing utility to confirm your result.